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Differential calculus multiple choice questions and answers

This set of Differential Calculus Multiple Choice Questions & Answers focuses on "Polar Curves". 1. For the below mentioned figure the angle between radius vector (op) and tangent to the polar curve where $r=f(\theta)$ has the one among the following relation? a) $\tan\omega = \frac{\tan\theta + \tan\theta}{1 - \tan\theta \tan\theta}$ b) $\tan\omega = \frac{\tan\theta - \tan\theta}{1 + \tan\theta \tan\theta}$ c) $\tan\omega = r(\frac{dr}{d\theta})$ d) $\tan\omega = \tan\theta + \tan\theta$ View AnswerAnswer: a Explanation: From the fig (1) $\omega = \theta + \phi$ (triangle rule & $\hat{o} = \phi$) $\tan\omega = \tan(\theta + \phi)$ $\tan\omega = \frac{\tan\theta + \tan\phi}{1 - \tan\theta \tan\phi}$ further more w.k.t $x = r\cos\theta$, $y = r\sin\theta$ But $\tan\omega = \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ (slope of the tangent TT') $\tan\omega = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dy}{d\theta}}{\frac{r\cos\theta + \frac{dr}{d\theta}\sin\theta}{r\sin\theta + \frac{dr}{d\theta}\cos\theta}} = \frac{r\sin\theta + \frac{dr}{d\theta}\cos\theta}{r\cos\theta + \frac{dr}{d\theta}\sin\theta} = \frac{r\sin\theta}{r\cos\theta} + \frac{\frac{dr}{d\theta}\cos\theta}{r\cos\theta} = \frac{r\sin\theta}{r\cos\theta} + \frac{\frac{dr}{d\theta}\cos\theta}{r\cos\theta} = \frac{r\sin\theta}{r\cos\theta} + \frac{r\cos\theta}{r\cos\theta} = \frac{r\sin\theta + r\cos\theta}{r\cos\theta} = \frac{r(\sin\theta + \cos\theta)}{r\cos\theta} = \frac{\sin\theta + \cos\theta}{\cos\theta} = \tan\theta + \frac{1}{\tan\theta}$ since x & y are functions of θ i.e. $\tan\omega = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dy}{d\theta}}{\frac{r\cos\theta + \frac{dr}{d\theta}\sin\theta}{r\sin\theta + \frac{dr}{d\theta}\cos\theta}} = \frac{r\sin\theta + \frac{dr}{d\theta}\cos\theta}{r\cos\theta + \frac{dr}{d\theta}\sin\theta} = \frac{r\sin\theta}{r\cos\theta} + \frac{\frac{dr}{d\theta}\cos\theta}{r\cos\theta} = \frac{r\sin\theta}{r\cos\theta} + \frac{r\cos\theta}{r\cos\theta} = \frac{r\sin\theta + r\cos\theta}{r\cos\theta} = \frac{r(\sin\theta + \cos\theta)}{r\cos\theta} = \frac{\sin\theta + \cos\theta}{\cos\theta} = \tan\theta + \frac{1}{\tan\theta}$(1) further more w.k.t $x = r\cos\theta$, $y = r\sin\theta$ But $\tan\omega = \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ (slope of the tangent TT') $\tan\omega = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{dy}{d\theta}}{\frac{r\cos\theta + \frac{dr}{d\theta}\sin\theta}{r\sin\theta + \frac{dr}{d\theta}\cos\theta}} = \frac{r\sin\theta + \frac{dr}{d\theta}\cos\theta}{r\cos\theta + \frac{dr}{d\theta}\sin\theta} = \frac{r\sin\theta}{r\cos\theta} + \frac{\frac{dr}{d\theta}\cos\theta}{r\cos\theta} = \frac{r\sin\theta}{r\cos\theta} + \frac{r\cos\theta}{r\cos\theta} = \frac{r\sin\theta + r\cos\theta}{r\cos\theta} = \frac{r(\sin\theta + \cos\theta)}{r\cos\theta} = \frac{\sin\theta + \cos\theta}{\cos\theta} = \tan\theta + \frac{1}{\tan\theta}$(2) from (1) & (2) $\tan\theta = \frac{\tan\theta + \tan\phi}{1 - \tan\theta \tan\phi} = r(\frac{dr}{d\theta})$. 2. The angle between Radius vector $r=a(1-\cos\theta)$ and tangent to the curve is ϕ given by _____ a) $\phi = (\frac{\pi}{2})$ b) $\phi = \pi$ c) $\phi = (-\frac{\pi}{2})$ d) $\phi = 0$ View AnswerAnswer: a Explanation: $r = a(1-\cos\theta)$ taking logarithms on both sides we get, $\log r = \log(a(1-\cos\theta))$ differentiating w.r.t θ we get, $\frac{1}{r} \frac{dr}{d\theta} = 0 + \frac{1}{a(1-\cos\theta)} \frac{d}{d\theta}(a(1-\cos\theta))$ $\frac{1}{r} \frac{dr}{d\theta} = \frac{1}{a(1-\cos\theta)} \frac{d}{d\theta}(a(1-\cos\theta)) = \frac{1}{a(1-\cos\theta)} \frac{d}{d\theta}(a(-\cos\theta)) = \frac{1}{a(1-\cos\theta)} a(\sin\theta) = \frac{\sin\theta}{a(1-\cos\theta)}$(1), but $\cot\phi = \frac{1}{\frac{dr}{d\theta}}$(2) From (1)&(2) $\phi = (\frac{\pi}{2})$. 3. Angle of intersection of two polar curves is equal to the angle between the tangents drawn at the point of intersection of the two curves then What is the condition for the two curves intersecting orthogonally for the below mentioned figure? a) $\tan\phi_1 \cdot \tan\phi_3 = 1$ b) $\tan\phi_1 \cdot \tan\phi_3 = -1$ c) $\tan(\phi_1 + \phi_3) = (\frac{1}{\sqrt{2}})$ d) $\tan(\phi_1 - \phi_3) = (\frac{1}{\sqrt{2}})$ View AnswerAnswer: b Explanation: From the figure & according to orthogonal principle $|\phi_1 - \phi_3| = (\frac{\pi}{2})$ further $\phi_1 = (\frac{\pi}{2}) + \phi_3$taking tangent of the angle $\tan\phi_1 = \tan(\frac{\pi}{2} + \phi_3) = -\cot\phi_3 = -\frac{1}{\tan\phi_3}$ $\tan\phi_1 \cdot \tan\phi_3 = -1$. 4. Angle of intersection between two polar curves given by $r = a(1+\sin\theta)$ & $r = a(1-\sin\theta)$ is given by _____ a) $(\frac{\pi}{4})$ b) $(\frac{\pi}{2})$ c) π d) 0 View AnswerAnswer: b Explanation: $r = a(1+\sin\theta)$: $r = a(1-\sin\theta)$ taking logarithm on both the equations $\log r = \log(a(1+\sin\theta))$: $\log r = \log(a(1-\sin\theta))$ differentiating on both side we get $\frac{1}{r} \frac{dr}{d\theta} = \frac{1}{a(1+\sin\theta)} \frac{d}{d\theta}(a(1+\sin\theta)) = \frac{1}{a(1+\sin\theta)} \frac{d}{d\theta}(a(\cos\theta)) = \frac{1}{a(1+\sin\theta)} a(-\sin\theta) = \frac{-\sin\theta}{a(1+\sin\theta)}$(1) & $\frac{1}{r} \frac{dr}{d\theta} = \frac{1}{a(1-\sin\theta)} \frac{d}{d\theta}(a(1-\sin\theta)) = \frac{1}{a(1-\sin\theta)} \frac{d}{d\theta}(a(\cos\theta)) = \frac{1}{a(1-\sin\theta)} a(-\sin\theta) = \frac{-\sin\theta}{a(1-\sin\theta)}$(2) from (1)&(2) $\cot\phi_1 = \frac{\frac{-\sin\theta}{a(1+\sin\theta)}}{\frac{-\sin\theta}{a(1-\sin\theta)}} = \frac{a(1-\sin\theta)}{a(1+\sin\theta)} = \frac{1-\sin\theta}{1+\sin\theta}$(3) $\cot\phi_2 = \frac{\frac{-\sin\theta}{a(1-\sin\theta)}}{\frac{-\sin\theta}{a(1+\sin\theta)}} = \frac{a(1+\sin\theta)}{a(1-\sin\theta)} = \frac{1+\sin\theta}{1-\sin\theta}$(4) above is the condition of orthogonality of two polar curves thus $|\phi_1 - \phi_2| = (\frac{\pi}{2})$. 5. One among the following is the correct explanation of pedal equation of an polar curve, $r=f(\theta)$, $p=r\sin(\phi)$ (where p is the length of the perpendicular from the pole to the tangent & ϕ is the angle made by tangent to the curve with vector drawn to curve from pole) is _____ a) It is expressed in terms of p & θ only b) It is expressed in terms of p & ϕ only c) It is expressed in terms of r & θ only d) It is expressed in terms of p & r only View AnswerAnswer: d Explanation: It is expressed in terms of p & r only where $p=r\sin(\phi)$ & $\tan\phi = \frac{r}{p}$ or after solving we get direct relationship between p & r as $\frac{p^2}{r^2} = \frac{1}{1 + \tan^2\phi}$. 6. The pedal Equation of the polar curve $r=n\cos\theta$ is given by _____ a) $rn=pan$ b) $rn-1=pan+1$ c) $rn+1=pan$ d) $rn+1=pan$ View AnswerAnswer: d Explanation: Taking logarithm for the given curve we get $n \log r = n \log a + \log(\cos\theta)$ differentiating w.r.t θ , we get $\frac{1}{r} \frac{dr}{d\theta} = \frac{n}{a} \frac{-\sin\theta}{\cos\theta}$(1) from the eqn w.k.t $p=r\sin\phi$ substituting from (1) $p = r\sin\phi = r\cos(n\theta)$, but we have $rn = a\cos\theta$ hence dividing them we get $\frac{rn}{a\cos(n\theta)} = \frac{1}{\cos(n\theta)}$ $rn+1=pan$. 7. The length of the perpendicular from the pole to the tangent at the point $\theta=(\frac{\pi}{2})$ on the curve. $r=a\sec(\frac{\pi}{2})$ is _____ a) $p=\frac{1}{\sqrt{2a}}$ b) $p=\frac{1}{2\sqrt{2a}}$ c) $p=2\sqrt{2a}$ d) $p=4a$ View AnswerAnswer: c Explanation: Taking Logarithm on both side of the polar curve we get $\log r = \log a + 2\log\sec(\frac{\pi}{2})$ differentiating w.r.t θ we get $\frac{1}{r} \frac{dr}{d\theta} = \frac{1}{a} \frac{2\sec^2(\frac{\pi}{2})}{\sec(\frac{\pi}{2})} = \frac{2\sec^2(\frac{\pi}{2})}{\sec(\frac{\pi}{2})} = 2\sec(\frac{\pi}{2})$(1) $\cot\phi = \frac{\frac{1}{r} \frac{dr}{d\theta}}{\frac{1}{r}} = \frac{2\sec^2(\frac{\pi}{2})}{\sec(\frac{\pi}{2})} = 2\sec(\frac{\pi}{2})$(2) $\phi = \frac{\pi}{2}$(3) w.k.t length of the perpendicular is given by $p=r\sin\phi$ thus substituting ϕ value we get $p=r\sin(\frac{\pi}{2}) = r\cos(\frac{\pi}{2}) = \frac{1}{\sec(\frac{\pi}{2})}$(4) but $r=a\sec(\frac{\pi}{2})$, $r=a\sec^2(\frac{\pi}{2}) = 4a$(5) from (1) & (2) $p=\frac{1}{\sec(\frac{\pi}{2})} = 2\sqrt{2a}$. 8. Polar equations of the circle for the given coordinate (x,y) which satisfies the equation given by $(x-a)^2 + (y-b)^2 = r^2$ where (a,b) is the coordinates of the centre of the circle & r is the radius. a) $x=r\cos\theta$, $y=r\sin\theta$ b) $x=a+r\cos\theta$, $y=b+r\sin\theta$ c) $y=a+r\cos\theta$, $x=b+r\sin\theta$ d) $x=r\sin\theta$, $y=r\cos\theta$ View AnswerAnswer: b Explanation: option x = a + r cos θ , y = b + r sin θ satisfies the equation $(x-a)^2 + (y-b)^2 = r^2$ because LHS = $(a+r\cos\theta-a)^2 + (b+r\sin\theta-b)^2 = r^2(\cos^2\theta + \sin^2\theta) = r^2$ = RHS. 9. In an polar curve $r=f(\theta)$ what is the relation between θ & the coordinates (x,y) ? a) $\tan\theta = \frac{y}{x}$ b) $(1+\sin\theta) = \frac{y}{x}$ c) $(1+\sec^2\theta) = \frac{y^2}{x^2}$ d) $(1+\cos\theta) = \frac{y}{x}$ View AnswerAnswer: c Explanation: w.k.t for the polar curve $r=f(\theta)$ $x=r\cos\theta$, $y=r\sin\theta$ dividing them we get $\frac{y}{x} = \frac{r\sin\theta}{r\cos\theta} = \tan\theta$ squaring on both side $\frac{y^2}{x^2} = \tan^2\theta = (1+\sec^2\theta)$. Sanfoundry Global Education & Learning Series – Differential Calculus. To practice all areas of Differential Calculus, here is complete set of 1000+ Multiple Choice Questions and Answers. Participate in the Sanfoundry Certification contest to get free Certificate of Merit. Join our social networks below and stay updated with latest contests, videos, internships and jobs! Manish Bhojasia, a technology veteran with 20+ years @ Cisco & Wipro, is Founder and CTO at Sanfoundry. 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